

# Medium polarization in asymmetric nuclear matter

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The influence of the core polarization on the effective nuclear interaction of asymmetric nuclear matter is calculated in the framework of the induced interaction theory. The strong isospin dependence of the density and spin density fluctuations is studied along with the interplay between the neutron and proton core polarizations. Moving from symmetric nuclear matter to pure neutron matter the crossover of the induced interaction from attractive to repulsive in the spin singlet state is determined as a function of the isospin imbalance. The density range in which it occurs is also determined. For the spin triplet state the induced interaction turns out to be always repulsive. The implications of the results for the neutron star superfluid phases are shortly discussed.

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## I. INTRODUCTION

Recently the interest for the superfluid states of neutron stars (NS) has been reviving after the real time temperature measurements of CasA remnant [1]. Theoretical models, devised to explain the cooling of such a system, demand for accurate predictions of the pairing gaps[2]. In particular, the study was focussed on the  $^1S_0$  proton-proton (p-p) and the  $^3PF_2$  neutron-neutron (n-n) pairing in the NS core. The peculiar aspect of NS pairing is that the onset of superfluidity occurs at high nuclear density, where the  $\beta$ -stability condition imposes large isospin imbalance. The latter feature could in fact influence to a large extent the vertex corrections to be included in the gap equation. Whereas it was definitely established that the self-energy corrections suppress the pairing magnitude[3], to what extent the core polarization affects the pairing interaction is not yet sufficiently clarified. This effect has been studied for several years (an extensive bibliography is in Ref.[4]) in the framework of the induced interaction theory (IIT)[5, 6]. More recently, Ref.[7] considered the two extreme situations of pure neutron matter(PNM) and symmetric nuclear matter(SNM). It was found that the core polarization, that in PNM quenches the n-n  $^1S_0$  pairing gap, on the contrary enhances it in SNM. The main conclusion was that in SNM a large compensation occurs between self-energy and vertex corrections, at least in the  $^1S_0$  n-n pairing. More recently polarization effects based on the RPA limit were included into the interaction to calculate the  $^1S_0$  p-p pairing in  $\beta$ -stable nuclear matter[8] and the  $^3PF_2$  n-n pairing in pure neutron matter[9].

The present paper is intended to extend the study of Ref.[7] to asymmetric nuclear matter and to report new calculations of the particle-hole (ph) residual interaction and the polarization propagator, in the realistic context of  $\beta$ -stable nuclear matter suitable for application to the NS pairing. The main scope is to investigate the interplay between density fluctuations and spin density fluctuations in singlet and triplet spin states within the particle-particle (pp) coupling, and in particular the crossover from attractive to repulsive interaction due to the core polarization, when moving from SNM to PNM. The baryon density threshold  $\rho_c$ , at which such a mechanism disappears, is also determined. These pieces of information are essential to study the various NS superfluid states.

## II. FORMALISM

The induced interaction theory for symmetric nuclear matter is reviewed in Ref.[10] (see also Ref.[7] and references therein quoted). Its extension to asymmetric nuclear matter is straightforward. In this case the ph irreducible interaction  $\mathcal{J}_{\tau\tau'}^S$ , is still rotationally invariant in spin space, S being the total spin in ph coupling, but it is not in isospin space, i.e.  $\mathcal{J}_{nn}^S \neq \mathcal{J}_{pp}^S$ . In the IIT framework  $\mathcal{J}_{\tau\tau'}^S$  is determined by the equation diagrammatically depicted

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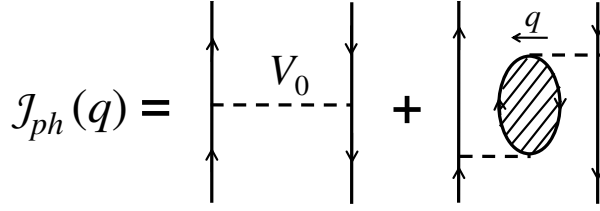


FIG. 1: Particle-hole induced interaction. The dotted line represents the direct term (G-matrix in the present approximation), the dashed lines represent the  $\mathcal{J}_{ph}$  itself, and  $q$  is the momentum transfer.

in Fig.1: The driving term (first term on the r.h.s. of Fig.1) is assumed to be approximated by G-matrix ; the induced term (second term on the r.h.s.) describes the interaction mediated by the medium excitations via the dressed polarization propagator (bubble in Fig.1). The vertex insertions represent the full ph irreducible interaction itself. The analytic expression of  $\mathcal{J}_{\tau\tau'}^S$  is given by the  $2 \times 2$  matrix equation in isospin space

$$\mathcal{J}^S(q) = G^S(q) + \mathcal{J}_i^S(q) \quad (1)$$

$$\mathcal{J}_i^0(q) = \frac{1}{2} \sum_{S'} (2S' + 1) \mathcal{J}^{S'}(q) \Lambda^{S'}(q) \mathcal{J}^{S'}(q) \quad (2)$$

$$\mathcal{J}_i^1(q) = \frac{1}{2} \sum_{S'} (-1)^{S'} \mathcal{J}^{S'}(q) \Lambda^{S'}(q) \mathcal{J}^{S'}(q), \quad (3)$$

where  $q \equiv (\vec{q}, \omega)$  is the energy-momentum transfer and  $\mathcal{J}_i$  denotes the induced interaction. This equation is solved in the Landau limit [6]. The polarization propagator  $\Lambda^S(q)$  is calculated as bubble series but, at variance with RPA, the vertex insertions between bubbles are expressed in terms of whole ph interaction  $\mathcal{J}^S$ [5]. In that limit this series can be summed up, resulting into a simple algebraic formula for the diagonal matrix elements

$$\Lambda_{\tau,\tau}^S(q) = \lambda_\tau(q) \frac{(1 + \lambda_{\tau'}(q) \mathcal{J}_{\tau',\tau'}^S(q))}{\mathcal{D}^S(q)} \quad (4)$$

and for the off-diagonal matrix elements

$$\Lambda_{\tau,\tau'}^S(q) = - \frac{\lambda_\tau(q) \lambda_{\tau'}(q) \mathcal{J}_{\tau,\tau'}^S(q)}{\mathcal{D}^S(q)} \quad (5)$$

$$1/\mathcal{D}^S = (1 + \lambda_\tau \mathcal{J}_{\tau,\tau}^S)(1 + \lambda_{\tau'} \mathcal{J}_{\tau',\tau'}^S) - \lambda_\tau \lambda_{\tau'} (\mathcal{J}_{\tau,\tau'}^S)^2, \quad (6)$$

where  $\tau \neq \tau'$ .  $\lambda_\tau(q)$  is the free polarization propagator[11] corrected by the mean-field effects (in the last line the  $q$  dependence is understood). Its explicit form is

$$\lambda_\tau(q) = Z_\tau^2 N_\tau \lambda\left(\frac{k}{k_F^\tau}, \frac{\omega}{\varepsilon_F^\tau}\right),$$

where  $\lambda(q)$  is the Lindhard function[12].  $N_\tau$  and  $Z_\tau$  are level density and quasi-particle strength at the Fermi surface, respectively, calculated within the extended Brueckner-Hartree-Fock (BHF) approximation (see Ref.[3]). It is worthwhile noticing that the bubble expansion is expected to be rapidly convergent at high density because  $Z^2 \ll 1$ .

### III. RESULTS AND DISCUSSION

As in Ref. [7], Eqs.(1)-(3) have been solved by means of the so called LNS potential, that is a Skyrme parametrization of the energy density functional calculated in the BHF approximation[13]. The difference from the pure Skyrme force is that the LNS parameters are derived from *ab initio* calculations rather than by fitting the empirical nuclear data. The LNS interaction enables a strong simplification of the formalism [14, 15].

The calculation of the ph induced interaction in nuclear matter ( $\mathcal{J}_i^S$ ) $_{\tau,\tau'}$  was performed in a wide density range at  $q = 0$ , and extended to non vanishing values of momentum transfer, as requested by the calculation of physical observables. In the Landau limit the  $q$ -range is restricted to  $0 \leq q \leq 2k_F^\tau(\rho)$ , being  $k_F^\tau$  the Fermi momentum

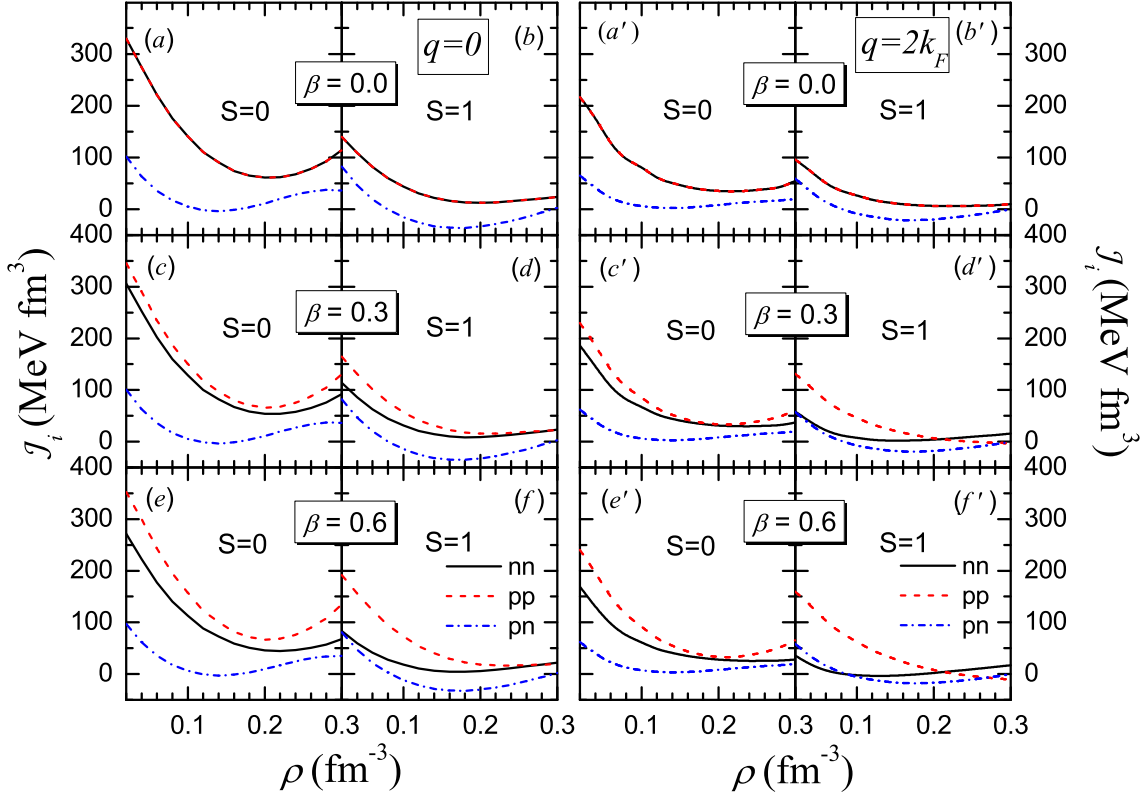


FIG. 2: Particle-hole induced interaction in nuclear matter vs. density for three values of  $\beta$  at  $q = 0$  (left) and  $q = 2k_F$  (right).

corresponding to the density  $\rho_\tau$ . In the present paper only  $q = 0$  and  $q = 2k_F$  will be discussed,  $k_F$  being the Fermi momentum corresponding to the total density.

The results are shown in Fig. 2, for the two values of the total spin  $S$  in ph coupling and the two limiting values of momentum transfer range,  $q = 0$  and  $q = 2k_F$ . Three values of the symmetry parameter  $\beta = (N - Z)/A$  have been chosen, including for comparison the symmetric case. At  $\beta = 0$  the curves corresponding to n-n and p-p matrix elements overlap, whereas at  $\beta > 0$  they split into two different components for the isospin symmetry breaking in asymmetric nuclear matter. The general trend is that  $\mathcal{J}_i^S$  is much larger at low density, as expected from the long-range character of the induced interaction. In addition, the isospin splitting  $(\mathcal{J}_i)_{pp}^S - (\mathcal{J}_i)_{nn}^S$  is increasing with  $\beta$ , whereas  $(\mathcal{J}_i)_{np}^S$  is almost insensitive. In r.h.s. of Fig. 2 the dependence on the momentum transfer is depicted. In our approximation only the kinematic dependence of the free polarization propagator is considered, that is monotonically decreasing with  $q$ . Therefore, as shown in Fig. 2,  $\mathcal{J}_i$  takes the smallest values at the upper limit  $q = 2k_F$  of the momentum transfer range.

In symmetric nuclear matter,  $\beta = 0$ , the effective interaction can be expressed in terms of the Landau-Migdal (LM) parameters [16]. On microscopic basis the induced interaction  $(\mathcal{J}_i)$  is the correction to the mean field prediction: the  $S=0$  curves correct the LM parameters  $F_{nn} = F_{pp}$  and  $F_{np} = F_{pn}$ , the  $S=1$  curves correct the LM parameters  $G_{nn} = G_{pp}$  and  $G_{np} = G_{pn}$ . It is worthwhile noticing that, in the induced interaction theory, the low-density instability region of the equation of state, corresponding to negative compression modulus, disappears at any  $\beta$ , as it can be easily checked in the calculation of the LM parameters. This is an old standing result at  $\beta = 0$  [10].

The nuclear induced interaction has also been calculated in the spin states of neutron-rich matter ( $N > Z$ ) in pp coupling, for the sake of application to the NS pairing. The n-n and p-p matrix elements in the spin singlet state are given by

$$(\mathcal{J}_i^{1S_0})_{\tau,\tau} = \frac{1}{2}[(\mathcal{J}_i)_{\tau,\tau}^0 - 3(\mathcal{J}_i)_{\tau,\tau}^1], \quad (7)$$

and in the spin-triplet pp state

$$(\mathcal{J}_i^{3PF_1})_{\tau,\tau} = \frac{1}{2}[(\mathcal{J}_i)_{\tau,\tau}^0 + (\mathcal{J}_i)_{\tau,\tau}^1]. \quad (8)$$

Moving from SNM to PNM the spin singlet interaction is mainly driven by the interplay between density fluctuations ( $S = 0$ ) and spin density fluctuations ( $S = 1$ ). The multiplicity of the latter is expected to play the major role considering that the two contributions are of the same order of magnitude[17]. The spin triplet interaction is only driven by the interplay between neutron and proton core polarizations.

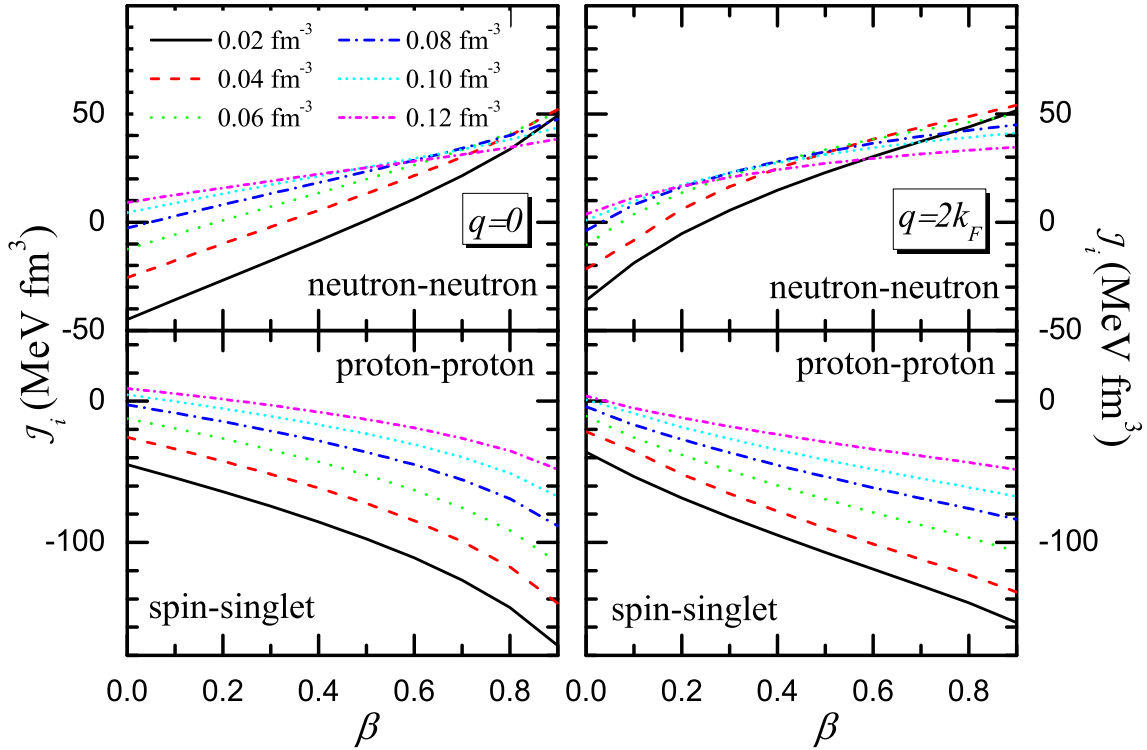


FIG. 3: Particle-particle induced interaction in the spin singlet state at  $q = 0$  (left) and  $q = 2k_F$  (right). The upper (lower) panels refer to neutron (proton) interaction.

The results for the spin singlet state are displayed in Fig. 3 as a function of the symmetry parameter for different densities of neutron-rich matter. We notice that the n-n effective interaction is attractive on the side of symmetric nuclear matter, ( $\beta = 0$ ), indicating the dominance of spin-density excitations over the density excitations, whereas it is the other way around on the side of almost pure neutron matter ( $\beta = 0.9$ ). Interesting is the value of the symmetry parameter  $\beta_c$ , where the two effects balance each other, marking the crossover from attractive to repulsive interaction. For asymmetries close to  $\beta_c$  the effective interaction is vanishing small, and, at variance with the symmetric nuclear matter, no compensation is allowed between self-energy and vertex corrections [7]. The value of the crossover asymmetry  $\beta_c$  is depending on density. There is density threshold  $\rho_c$ : for  $\rho > \rho_c$  the induced interaction is repulsive at any asymmetry. In the present estimate for  $\rho_c$  is about  $0.09 \text{ fm}^{-3}$ . In the case of the p-p effective interaction the situation is quite different, because in this case the neutron core polarization is attractive and the proton core polarization is repulsive. Therefore, increasing asymmetry, the former becomes more and more dominant, resulting into an increasingly attractive strength of the induced interaction. In the p-p case the critical density  $\rho_c$  is the lower limit for the existence of the isospin crossover.

The results at  $q = 2k_F$  are depicted in r.h.s. of Fig. 3. From the kinematic behavior of the free propagator it turns out that, increasing the momentum transfer, the crossover is pushed to lower values of both asymmetry and density in the n-n case but rapidly disappears in the case of protons.

The p-p effective interaction in spin singlet state was studied in Ref. [8] for nuclear matter in  $\beta$ -stable regime. At  $\rho = 0.2 \text{ fm}^{-3}$  it was found a value of  $-14.2 \text{ MeV fm}^3$  to be compared with the value  $-1.2 \text{ MeV fm}^3$  from the present study. The deviation by one order of magnitude should not be considered too large, taking into account the different approximations adopted in the two approaches, in particular, the missing three-body force in the Ref. [8] calculation.

The results for the triplet case are depicted in Fig. 4. In such a case there no interplay between density and spin density fluctuations so that no any crossover is possible. The isospin dependence is easily understood by the competition between neutron and proton polarization: moving from SNM to PNM the n-n  $J_i$  monotonically decreasing and p-p  $J_i$  is monotonically increasing. The main feature is that the induced interaction is always repulsive, the repulsion being attenuated by finite momentum transfer, as shown in Fig. 4 (left). This property may eventually explain why in finite nuclei it is found that strength of T=0 pairing is never stronger than T=1

strength in spite of the fact that the bare  $T=0$  force yields a much stronger gap than the  $S=0$  one.

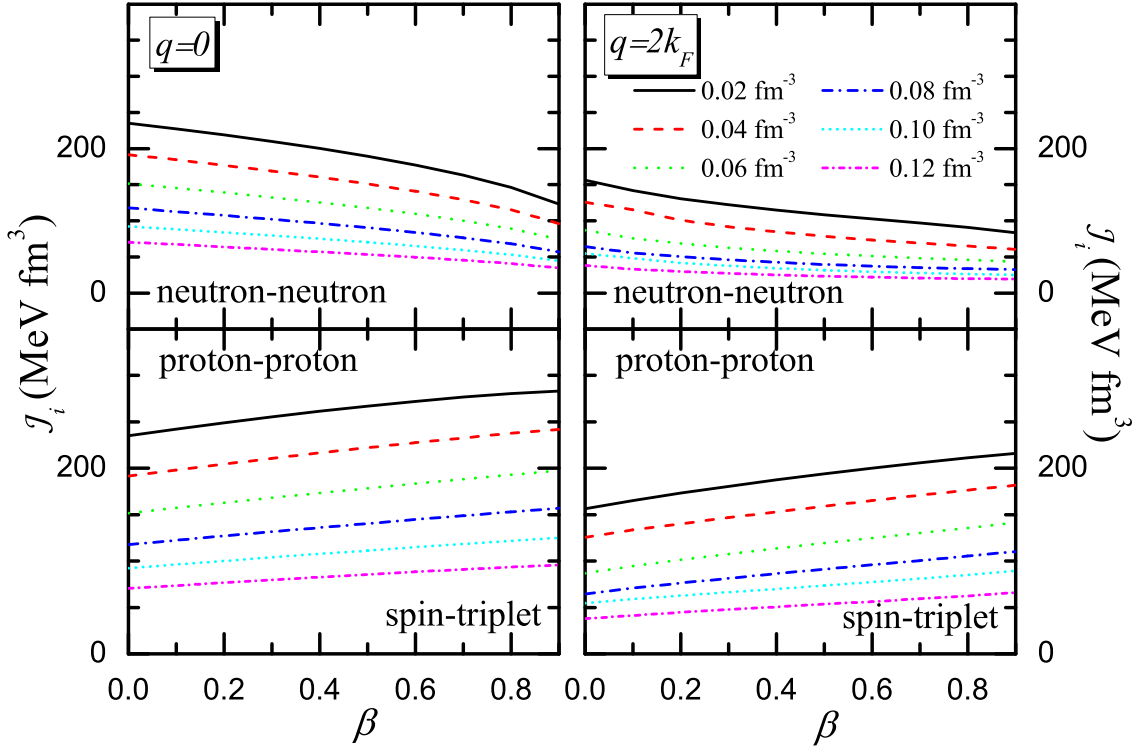


FIG. 4: Particle-particle induced interaction in the spin triplet state at  $q = 0$  (left) and  $q = 2k_F$  (right). The upper (lower) panels refer to neutron (proton) interaction.

In conclusion, from the calculations of the induced interaction one should expect the core polarization to have a deep influence on the pairing interaction in  $\beta$ -stable nuclear matter and, in particular, on the three superfluid states supposed to exist in NS. In the case of the low-density n-n  $^1S_0$  pairing, located in the NS crust, the induced interaction turns out to be repulsive, as shown in the Fig. 3 for spin singlet states, because the crust is a very neutron-rich state. Therefore, the gap is expected to be reduced not only by the self-energy corrections, but also by the induced interaction. On the other hand, in the case of the high density  $^3PF_2$  n-n pairing, located in the NS inner core, the strong core polarization, that is always repulsive in spin triplet states, should suppress completely the high density n-n pairing, at variance with Ref. [9]. On the contrary, in the case of  $^1S_0$  p-p pairing, where low-density proton fraction is embedded into high-density neutron matter, the neutron core induces a strong attractive enhancement on the p-p Cooper pairs so to compete with the self-energy suppression.

Therefore, after the introduction in the gap equation of the self-energy corrections of  $\beta$ -stable nuclear matter [3], the inclusion of the induced interaction on the same footing should rise the theoretical study of NS superfluidity to a quite satisfactory stage. Such calculations are in progress.

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